Period 3, April 7, 2025

$\frac{dy}{dx}=2$ $\quad 1\cdot y + x\cdot\frac{dy}{dx}=0$

14 If $y = 2x - 8$, what is the minimum value of the product xy?

(A) -16 (B) -8 (C) -4 (D) 0 (E) ~~2~~

$2x-8+x\cdot 2=0$

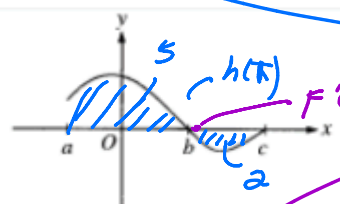
$4x-8=0$

$x=2$

$y=2(2)-8=-4$

$2\cdot -4=-8$

16

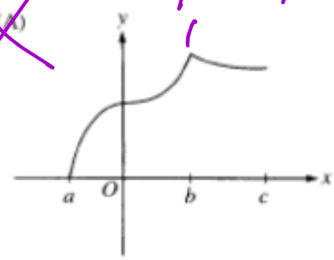


Not The Same

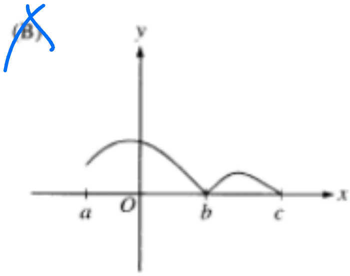
Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

~~(A)~~

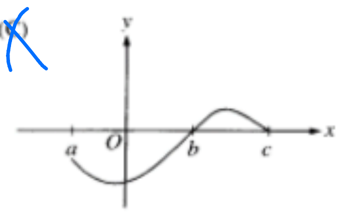
$F'(b)=\phi$



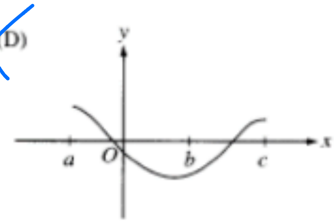
~~(B)~~



~~(C)~~

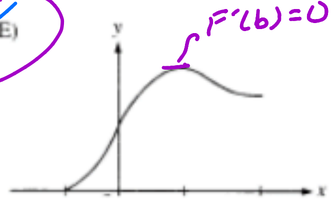


~~(D)~~



(E) ✓

$F'(b)=0$



19 If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

$F'(x) = (x+2)(x-2)$ negative

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$. ✓

(C) f has relative minima at $x = -2$ and at $x = 2$. x

(D) f has relative maxima at $x = -2$ and at $x = 2$. x

(E) It cannot be determined if f has any relative extrema. X

Test (-3, 0, 3)

min max

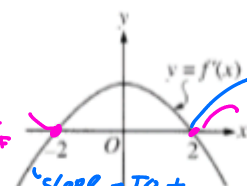
- - - - (+ + + + + + + (- - - - -

-3 -2 -1 0 1 2 3

Max slope + To -

min slope - To +

2



F(-2) Has a Slope of zero

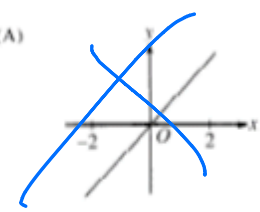
slope - To + min

slope + To - Max

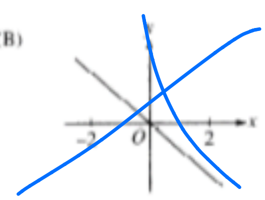
F(2) Has a Slope of zero

The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

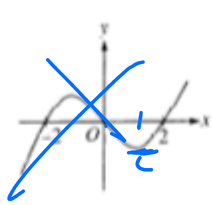
(A)



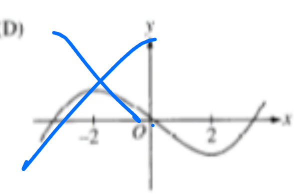
(B)



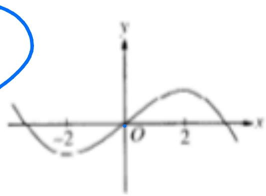
(C)



(D)



(E)

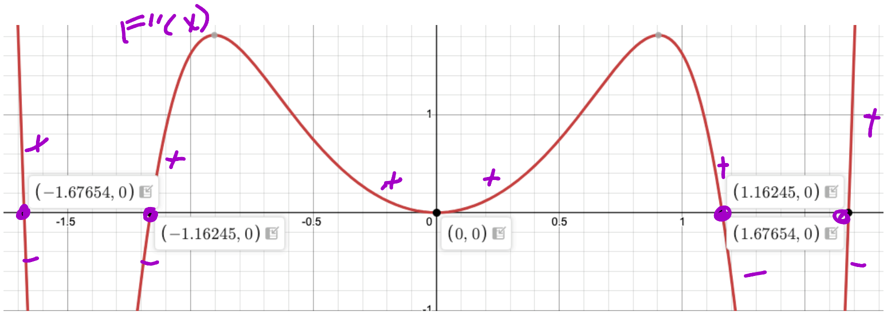


*15. Let $f(x)$ be the function with the derivative defined by $f'(x)=\sin(x^3)$ on the interval $-1.8<x<1.8$.
How many inflection points does $f(x)$ have on this interval?

concavity must change

(A) two (B) three (C) four (D) five (E) six

$F''(x)=\left[\cos(x^3)\right]\cdot\underbrace{3x^2}_{\text{always positive}}$



13 The graph of the function $y=x^3+6x^2+7x-2\cos x$ changes concavity at $x=$

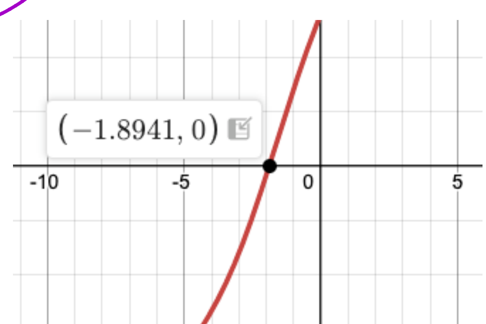
(A) −1.58 (B) −1.63 (C) −1.67 (D) −1.89 (E) −2.33

$\frac{dy}{dx}=3x^2+12x+7-2(-\sin x)=3x^2+12x+7+2\sin x$

$\frac{d^2y}{dx^2}=6x+12+2\cos x$

Graph

$y=6x+12+2\cos x$



2014

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

$F'(x)=0$

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers. $x=1$

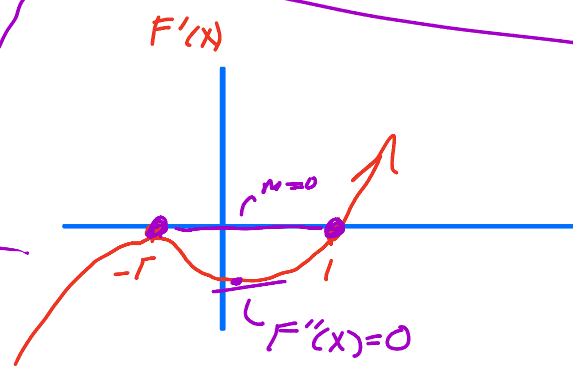
slope − TO +

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$. Rolles, MVT

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

(b) Rolles Theorem



(c) $h(x) = \ln(F(x))$

$h'(x) = \frac{1}{F(x)} \cdot F'(x)$

$h'(3) = \frac{1}{F(3)} \cdot F'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$

$y = \ln F(x) \Rightarrow y = \ln u$

$u = F(x)$ $\quad \frac{dy}{du} = \frac{1}{u}$

$\frac{du}{dx} = F'(x)$

$\frac{du}{dx} \cdot \frac{dy}{du} = F'(x) \cdot \frac{1}{u}$

$\frac{dy}{dx} = \frac{F'(x)}{F(x)}$

$\int_{-2}^{3} f'(g(x))g'(x)\,dx = \int F'(u) \cdot du$

$g(x) = u$

$g'(x)\,dx = du \Rightarrow$

$F(u)$ $\quad F(g(x)) \Big[_{-2}^{3}$

$F(g(3)) - F(g(-2)) \Rightarrow F(1) - F(-1) = 2 - 8 = -6$

2016 (calc)

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

$S(4) = 2$

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(T) = 1 + 2\sin\left(\frac{T^2}{2}\right)$ chain Rule

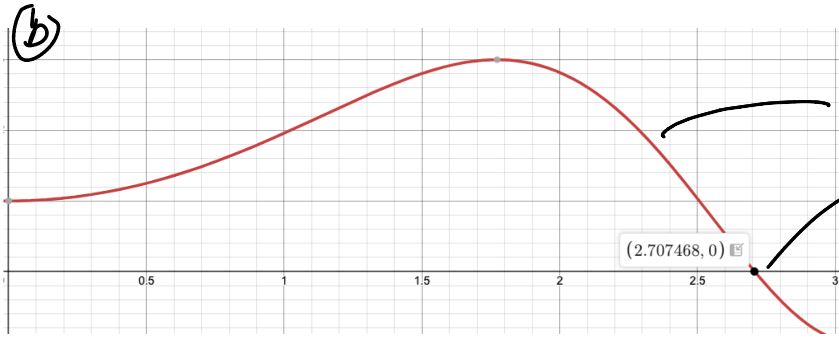
$a(T) = \left[2\cos\left(\frac{T^2}{2}\right)\right] \cdot T = 2T\cos\frac{T^2}{2}$

$v(4) = 1 + 2\sin 8 = 2.9787$

$a(4) = 2 \cdot 4\cos 8 = -1.164$

Slowing down, different signs

(b)



graph of $v(T)$

crosses zero so changes direction

(c) Find the position of the particle at time $t = 0$. $= \int_4^0 v(T)\,dT$

The particle is at position $x = 2$ at time $t = 4$.

$2 + \int_4^0 v(T)\,dT = 2 - 5.815 = -3.815$

$$y = \int_4^0 \left[1 + 2\sin\left(\frac{x^2}{2}\right)\right] dx$$

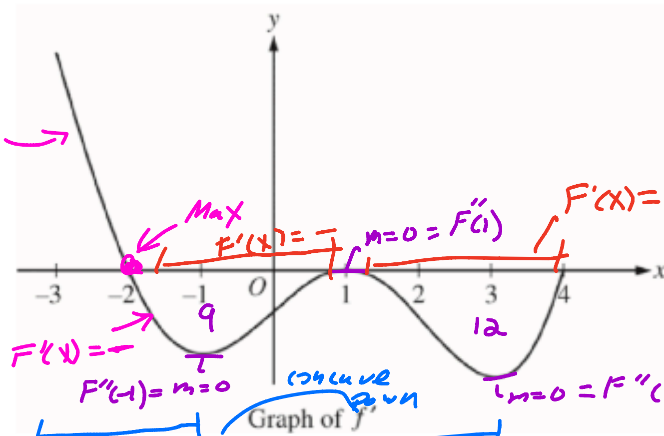
$= -5.81502683991$

(d) $\int_0^3 |v(t)|\,dt$

$y = \int_0^3 \left|\left[1 + 2\sin\left(\frac{x^2}{2}\right)\right]\right| dx$

$= 5.30119834688$

$F'(x) = +$



5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

slope from + to −

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

Slope of $F'(x) = F''(x) = -$ $\qquad$ $F'(x) = -$

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(b) Concave: down, up, down, up ($-3\ \ -2\ \ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4$)

increase/decrease: in, decrease, decrease ($-3\ \ -2\ \ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4$)

$(-2, -1) \cup (1, 3)$

(c) $F''(x) = 0 \quad x = -1, 1, 3$

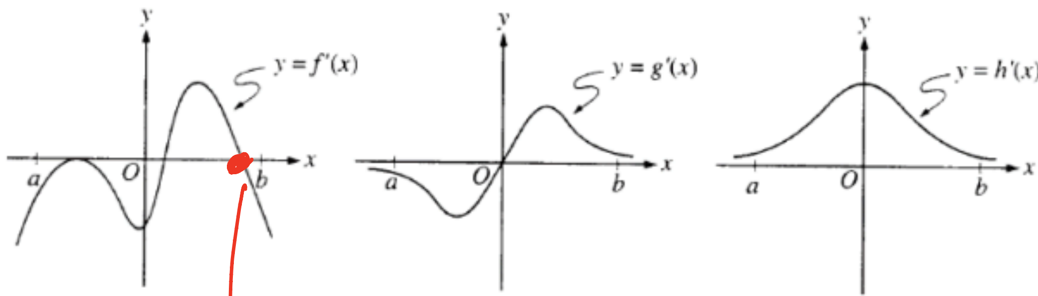
inflection points

change concavity

(d) $F(4) = 3 + \int_1^4 F'(x)\,dx = 3 + -12 = -9$

$F(-2) = 3 + \int_1^{-2} F'(x)\,dx = 3 + 9 = 12$

17



The graphs of the derivatives of the functions *f*, *g*, and *h* are shown above. Which of the functions *f*, *g*, or *h* have a relative maximum on the open interval $a < x < b$?

(A) *f* only
(B) *g* only
(C) *h* only
(D) *f* and *g* only
(E) *f*, *g*, and *h*

F'(x) or g'(x) or h'(x) must go From positive to negative

18

The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

$$0 = \frac{\cos^2 x}{x} - \frac{1}{5}$$

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven